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On the Influence of Momentum Conservation  
upon the Scaling Behaviour of Factorial Moments  
in High Energy Multiparticle Production

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**Abstract**

The experimental results on the falling down of scaled factorial moments in azimuthal variable  $\phi$  is studied in some detail. It is shown that this phenomenon may be referred to the influence of transverse momentum conservation. The existing experimental data from DELPHI, EMU08, NA22 and UA1 are successfully explained. Various methods are proposed to partly eliminate this influence and rule out the 'falling down' of factorial moments.

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Since the last decade the anomalous scaling of scaled factorial moments (SFM's) or intermittency[1] has been studied extensively[2] with the aim of exploring the possible existence of dynamical fluctuation or multifractal structure of multiparticle spectrum in high energy collision processes. The corresponding experiments have been performed in various kind of collisions — from  $e^+e^-$ , hadron-hadron, hadron-nucleus to nucleus-nucleus. Plenty of new and interesting experimental results have been put in front of us. Various efforts[3, 4, 5] have been taken to understand the physics behind these experimental findings.

In this respect, it merits attention that, in almost all the cases, the logarithm of SFM's versus that of the cell width  $\delta$  rises as the decreasing of  $\delta$  (or as the increasing of partition number  $M$  of phase space, where  $M = \Delta/\delta$ ,  $\Delta$  is the total phase space region in consideration), when  $M$  is not very big. There is, however, a noticeable exception, i.e. the logarithm of SFM's with azimuthal angle  $\phi$  as variable fall down as the increasing of  $\ln M$  in the first few points[6, 7, 8, 9], cf. Fig.1. This prevents us from properly understanding the physics behind the scaling behaviour of SFM's found in experiments.

The aim of this paper is to study this phenomenon in some detail. We will show that the falling down of SFM's in azimuthal angle  $\phi$  may be referred to the influence of momentum conservation (MmCn) constraint in the collision processes. Various methods will be implemented to partly eliminate this influence and explore the inherent scaling property of SFM's.

As is well known[1], intermittency refers to the power-law behaviour of the scaled factorial moments  $F_q$  as function of a cell width  $\delta$  of the phase space, in which the hadron multiplicity is measured.  $F_q$  is defined by

$$F_q = \frac{1}{M} \sum_{m=1}^M \frac{\langle n_m(n_m-1) \cdots (n_m-q+1) \rangle}{\langle n_m \rangle^q}, \quad (1)$$

where  $M$  is the number of cells in horizontal average,  $n_m$  is the multiplicity in  $m$ th  $\delta$ -cell in a high energy collision event, and  $\langle \rangle$  denotes vertically averaging over many events.

However, it should be taken into account that the total momenta of collision system must keep invariant in the space-time evolution process of the high energy collisions. This momentum conservation constraint may create some correlation between the multiplicity  $n_m$  of distinct phase space cells. But what is the influence of this correlation due to MmCn constraint upon the values of  $F_q$  for different partition number  $M$ ? In order to attack the solution of this question more directly, Let us at first take the 1-dimensional (1D) momentum phase space as an example.

Let  $\Delta p$  denotes the total momentum district in consideration. The momentum  $p_i$  of each particle is restricted by the conservation condition  $\sum_{i=1}^n p_i = 0$ , where  $n$  is the total number of particle inside the region  $\Delta p$ .

Firstly, it is clear that MmCn has no influence on the value of  $F_q$  for  $M = 1$ , because  $F_q$  for  $M = 1$  depends only on the total number  $n$  of particles in the whole region  $\Delta p$  no matter how these particles are distributed, cf. Eq.( 1). More briefly,  $F_q$  is 'blind' to the different distributions of  $p$ , therefore it takes the same value regardless of whether there is or isn't MmCn.

When  $M = 2$  the influence of MmCn comes in force, diminishing the value of  $F_q$ . This can be seen as the following. When  $M = 2$  the total region  $\Delta p$  is divided into two bins with positive and negative  $p$  respectively. Let  $P_+$  and  $P_-$  be the probabilities for a particle to lie in these two bins respectively. If there were neither dynamical fluctuation DF (i.e. if the probability did not fluctuate in itself) nor momentum conservation constraint, these two possibilities would be the same:

$$P_+ = P_- \quad (\text{in case of neither DF nor MmCn}).$$

In an individual event, the statistical fluctuation (SF) will make the multiplicity  $n_+$  and  $n_-$  of particles with positive and negative  $p$  fluctuate,

$$n_+ \neq n_- \quad (\text{randomly due to SF}).$$

Assuming SF to be of Bernouli or Poisson type, it can be eliminated by means of factorial moments method[1], resulting in a flat SFM versus  $M$ , i.e.

$$F_q^{(M=2)} = F_q^{(M=1)} \quad (\text{in case of neither DF nor MmCn}).$$

If there is dynamical fluctuation, the  $P_+$  and  $P_-$  themselves will be unequal in an individual event, resulting in much larger fluctuations in  $n_+$  and  $n_-$ , so that  $F_q$  will rise from  $M = 1$  to  $M = 2$ . On the other hand, The momentum conservation constraint  $\sum_{i=1}^n p_i = 0$  tends to separate particles to the positive and negative  $p$  regions, which reduces the difference between  $n_+$  and  $n_-$  caused by DF and/or SF, making  $F_q$  smaller than what would be expected without the restriction of MmCn.

Thus it is predicted that  $F_q$  should not rise so steeply as required by DF or even may fall down from the first point ( $M = 1$ ) to the second point ( $M = 2$ ) in contradiction to the flat SFM required by pure SF, because of the correlation effect between multiplicity  $n_+$  and  $n_-$  from MmCn.

In Fig.2a,b is shown the results of Monte Carlo simulation of a random cascading model with and without MmCn. In this model, the total region  $\Delta p$  is divided subsequently with the subdividing ratio  $\lambda = 2$ . The probability  $w_i$  for a subdivision is taken to be

$$w_i = \frac{1 + \alpha r}{\sum_{i=1}^{\lambda} (1 + \alpha r_i)}, \quad i = 1, 2, \quad (2)$$

where  $\alpha$  is the parameter of fluctuation strength,  $0 \leq \alpha \leq 1$ ;  $r_i$  is a random number in the interval  $[-1, 1]$ . After  $\nu$  steps, the probability in  $k$ th subdivided bin ( $k = \{i_1, i_2, \dots, i_\nu\}$ ) is

$$p(k) = w_{i_1}^{(1)} w_{i_2}^{(2)} \dots w_{i_\nu}^{(\nu)}. \quad (3)$$

The  $n$  particles are then assigned to the subdivided bins according to the Bernoulli distribution

$$P(n_1, n_2, \dots, n_K) = \frac{n!}{n_1! n_2! \dots n_K!} p(1)^{n_1} p(2)^{n_2} \dots p(K)^{n_K}, \quad (4)$$

where  $K = 2^\nu$  is the total number of bins. Taking randomly the momenta of the  $n_k$  particles in the  $k$ th bin ( $k = 1, 2, \dots, K$ ), a Monte Carlo sample of particle momenta is obtained.

In the calculation we have taken the model parameter  $\alpha = 0.5$ , the cascading has been performed up to  $\nu = 7$ th step.

In this model the dynamical fluctuation is taken into account up to the  $\nu$ th step of random cascading, i.e. up to the bin width as small as  $\delta p = \Delta p / 2^\nu$ . When  $\nu$  is large,  $\delta p$  is very small, the neglecting of further DF will cause trivial error in the final results.

From the construction of model the resulting SFM's have the property of anomalous scaling

$$F_q(M) \propto M^\varphi, \quad (5)$$

so that the log-log plot of  $\ln F_q$  versus  $\ln M$  is a straight line pointing upward for the point of  $M = 2^{\nu 1}$ , cf. Fig.2a.

In the above model there is, of course, not any restriction from MmCn in the resulting sample, i.e.  $\sum_{i=1}^n p_i \neq 0$ . We use a simple method to put in the constraint of MmCn. Let

$$p_c = \frac{1}{n} \sum_{i=1}^n p_i, \quad (6)$$

and shift the momentum of every particle in the event by an amount  $p_c$

$$p'_i = p_i - p_c. \quad (7)$$

Then the new momenta  $p'_i$  obey the conservation law,  $\sum_{i=1}^n p'_i = 0$ . This transform does effect the distribution of  $p$ , cf. Fig.3, but not very much, and at the same time the fractal structure existing in the original sample retains in the new one. Using the cumulative variable as usual[11], the distribution becomes flat again and the SFM's can be calculated.

The results are shown in Fig.2b. It can be seen in the figure that the first point, corresponding to  $M = 1$  has the same height as that of Fig.2a while the second point,  $M = 2$ , is lower than that of Fig.2a due to MmCn, resulting in a falling down of  $\ln F_2$  from the first to the second point, confirming the prediction given above. When  $M$  increases further,  $\ln F_2$  increases again due to dynamical fluctuation.

Thus we see that the falling down of  $F_q$  at the first few points is due to putting in force of the MmCn effect. If we take the positive and negative  $p$  particles as particles of two "events", calculating SFM's in these "events" separately and then take the vertical average over them, the falling down phenomenon will disappear. It is really the case in our model, cf. Fig.2c.

On the other hand, if we make a cut and consider only the particles with  $|p| < p^{\text{cut}}$ , then the remaining particles will no longer be restricted so strongly by MmCn and the falling down phenomenon may gradually disappear. In Fig.2d is shown the result of our model with  $p^{\text{cut}} = 0.8$ , a straight line is recovered.

Let us now turn to the more realistic case of 3-dimensional (3D) phase space. It is noticeable that the momentum conservation constraint of sample is very different in transverse and longitudinal phase space. Generally, not all final fragments of high energy collisions are

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<sup>1</sup>The 'chain effect' may appear in the plot of  $\ln F_q$  versus  $\ln M$  if the continuously diminishing scale is taken in this kind of random cascading model[10].

considered in the intermittency experiments. In particular, the leading particles, which take away in the average 50% of longitudinal momenta, are neglected. Therefore, the restriction of longitudinal momentum conservation on the particles analysed is quite weak. In fact, the experimental data of SFM's in longitudinal phase space (e.g. rapidity  $y$  or pseudorapidity  $\eta$ ) do not fall down in the plot of  $\ln F_q \sim \ln M(y)$ , cf. Ref.[2].

On the other hand, the constraint effect of MmCn in transverse plan is a little different between the samples from the different experiments. It depends on the distinct experimental conditions, i.e. the analysed phase space district cover both the central and fragmental regions, or only the central region, or just part of the central region.

There is usually no cut in transverse momentum in the fixed target experiments and the  $e^+e^-$  collisions, and the transverse momenta of leading particles are almost zero. So if the produced particles in both central and fragmental region are considered (e.g. DELPHI collaboration[8]), the transverse momentum conservation must be satisfied very well.

In some intermittency experiments, only final state hadrons of central region are analysed. For example, the NA22 collaboration put a cut of rapidity  $|y| \leq 2$  in longitudinal direction[6] although there is no cut in the transverse directions. So the transverse momenta of final hadrons in fragmental region have been neglected. However, since many experimental finding had shown that the final particles in central and fragmental regions may come respectively from several relatively independent sources (or fire balls)[12], the transverse MmCn condition is also kept approximately for the particles of central region alone.

In UA1 experimental data[9], only a part of the particles in the central region are analysed. A pseudorapidity cut of  $|\eta| \leq 1.5$  is taken in addition to the beam pipe cut of  $p_t > 0.15\text{GeV}$  in transverse direction. So the influence of transverse MmCn in UA1 experimental data is not so strong, which will manifest itself by the weaker falling degree of SFM's in the plot of  $\ln F_q \sim \ln M(\phi)$ .

In order to reveal the influence of transverse MmCn upon the anomalous scaling behaviour of SFM's in 3D phase space, we generalize the above 1D cascading model with MmCn into 3D phase space. Constructing a 3D random cascading model in the space of  $0 \leq y \leq 1$ ,  $0 \leq p_t \leq 1$ ,  $0 \leq \phi \leq 2\pi$ , the results of Monte Carlo simulation are shown in the first column of Fig.5. The 3D  $(y, p_t, \phi)$  curve is a straight line as required by the cascading model, and the 1-, 2-dimensional SFM's get saturated, which is referred to the well known projection effect[13].

Now let us impose the transverse MmCn condition on the random cascading model. After all the vector momentum  $\vec{p}_i$  of the produced particles are determined in a simulated event, we shift the vector transverse momentum  $\vec{p}_{t_i}$  of every particle by an amount  $\vec{p}_{t_c} = \frac{1}{n} \sum_{i=1}^n \vec{p}_{t_i}$ , i.e.

$$\vec{p}'_{t_i} = \vec{p}_{t_i} - \frac{1}{n} \sum_{j=1}^n \vec{p}_{t_j}. \quad (8)$$

Then the transverse MmCn condition

$$\sum_{i=1}^n \vec{p}'_{t_i} = 0 \quad (9)$$

is satisfied.

The distribution function of rapidity  $y$  is, of course, not influenced by this transform (not shown in figure). The effect of the translation upon the distribution of  $p_t$  and  $\phi$  are shown in Fig.4. It can be seen from the figure that the  $\phi$  distribution is almost uninfluenced by the transform due to the axial symmetry. So the cumulative variable has to be used only for  $p_t$ .

The results of calculation with MmCn put in in this way are shown in the second column of Fig.5. It can be seen clearly that the falling down effect does take place for variable  $\phi$ . It is also interesting to notice that the minimum of SFM in the  $\ln F_2$  versus  $\ln M$  plot occurs at  $M(\phi) = 4$ , contradicting the 1D MC results with minimum location at  $M(\phi) = 2$ . This is as expected, because  $M(\phi) = 4$  means that the positive and negative transverse momenta in both directions (denoted by  $p_{ty}$  and  $p_{tz}$  respectively) are divided into different bins, while in the case of smaller  $M(\phi)$ ,  $M(\phi) = 2$  for example, the positive and negative transverse momentum in only one direction (say  $p_{ty}$ ) are separated. Therefore, in the case of  $M(\phi) = 2$ ,  $F_q$  is 'blind' to the different distributions in  $p_{tz}$  and the effect of reduction of DF by transverse MmCn is weaker correspondingly. This is why  $F_q(M = 1) > F_q(M = 2) > F_q(M = 4)$ .

If we consider the four quadrants as four different "events" and then take the vertical average, the falling down phenomenon will disappear, as can be seen in the result of model calculation shown in the third column of Fig.5.

If we impose a cut on  $p_t$  to weaken the influence of MmCn, and consider only particles with  $p_t < p_t^{\text{cut}}$ , then the falling down effect gets weakened too and disappear at about  $p_t^{\text{cut}} = 0.7$ , cf. Fig.6.

Thus we have successfully explained the experimental finding on the falling down of the first few points in the  $\ln F_q(\phi)$  versus  $\ln M(\phi)$  plot, cf. Fig.1. The transverse momentum conservation is satisfied in case that all the produced particles in both central and fragmental regions[7, 8] are considered, or approximately satisfied if only those in central region are considered. According to the above argument the minimum of  $\ln F_q$  versus  $\ln M(\phi)$  should occur at about  $M(\phi) = 4$ . It does occurs at about  $M(\phi) = 5$  in the experimental data in Fig.1a,b. On the contrary, in the  $p\bar{p}$  collider experiments of UA1 collaboration, only a part of the produced particles in central region are analysed, and there is a transverse  $|p_t| > 0.15\text{GeV}$  cut due to the existence of beam pipe. So the effect of transverse MmCn is much weaker, cf. Fig.1c and Fig.6c.

Although there is a difference between these two cases, i.e. in our model of Fig.6c we have imposed a cut at the upper side of  $p_t$ , while in UA1 experiment (Fig.1c), the cut is imposed at the lower side, the physics is the same, both are the relaxation of the restriction of transverse MmCn due to the particles carrying only a part of transverse momenta. It is noticeable that, besides the weakening of the falling down of SFM, this effect also makes its minimum move to a smaller value of  $M$ , locating at  $M < 4$ . This is the case both for the Monte Carlo model and for the experimental data, cf. Fig.1c and Fig.6c.

We conclude that in order to explore the physics behind the experimental phenomena on the scaling behaviour of scaled factorial moments (intermittency phenomena), the influence of transverse momentum conservation must be taken into account. From the constraint of

transverse momentum conservation, the falling-down of SFM with azimuthal angle variable in DELPHI, EMU08, NA22 and UA1 can be explained successfully.

Using the method of "quadrant analysis", i.e. taking the particles in different quadrant as different "event", or imposing a  $p_t$  cut, this influence can be partly eliminated, especially, the falling down of the first few points can be ruled out. So these methods are helpful to reveal the inherent scaling behaviours of a fractal system.

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## Figure Captions

**Fig.1** Scaled factorial moments with azimuthal angle as variable, showing the 'falling down' phenomenon. Data taken from Ref's.[6, 7, 9].

**Fig.2** The scaling behaviours of SFM's in one-dimensional random cascading model:  
(a), original cascading model without MmCn constraint;  
(b), the cascading model with MmCn considered;  
(c), the result of 'quadrant analysis' to the cascading model with MmCn;  
(d), the result of the cascading model with MmCn after a cut of  $|p| \leq 0.8$  is imposed.

**Fig.3** Distribution of  $p$  in 1D cascading model with and without MmCn.

**Fig.4** The influence of shift of transverse momenta upon the distribution function of  $p_t$  and  $\phi$  in 3D  $(y, p_t, \phi)$  phase space. The solid curve denote the distribution function after MmCn is inputted, and the dashed ones denote that without MmCn.

**Fig.5** The scaling behaviours of SFM's and their lower dimensional projection in 3D cascading model, where the first row is results of 3D  $(y - p_t - \phi)$  phase space, the second and third rows are that of 2D  $(y - \phi)$  and 1D  $(\phi)$  projections respectively  
(a), the original cascading model without MmCn (the first column);  
(b), the cascading model with MmCn (the second column);  
(c), the result of 'quadrant analysis' for the cascading model with MmCn (the third column).

**Fig.6** The results of the 3D cascading model with MmCn after a cut of  $|p| \leq p_t^{\text{cut}}$  is imposed, (a)-(d) correspond to different  $p_t^{\text{cut}}$  values respectively.



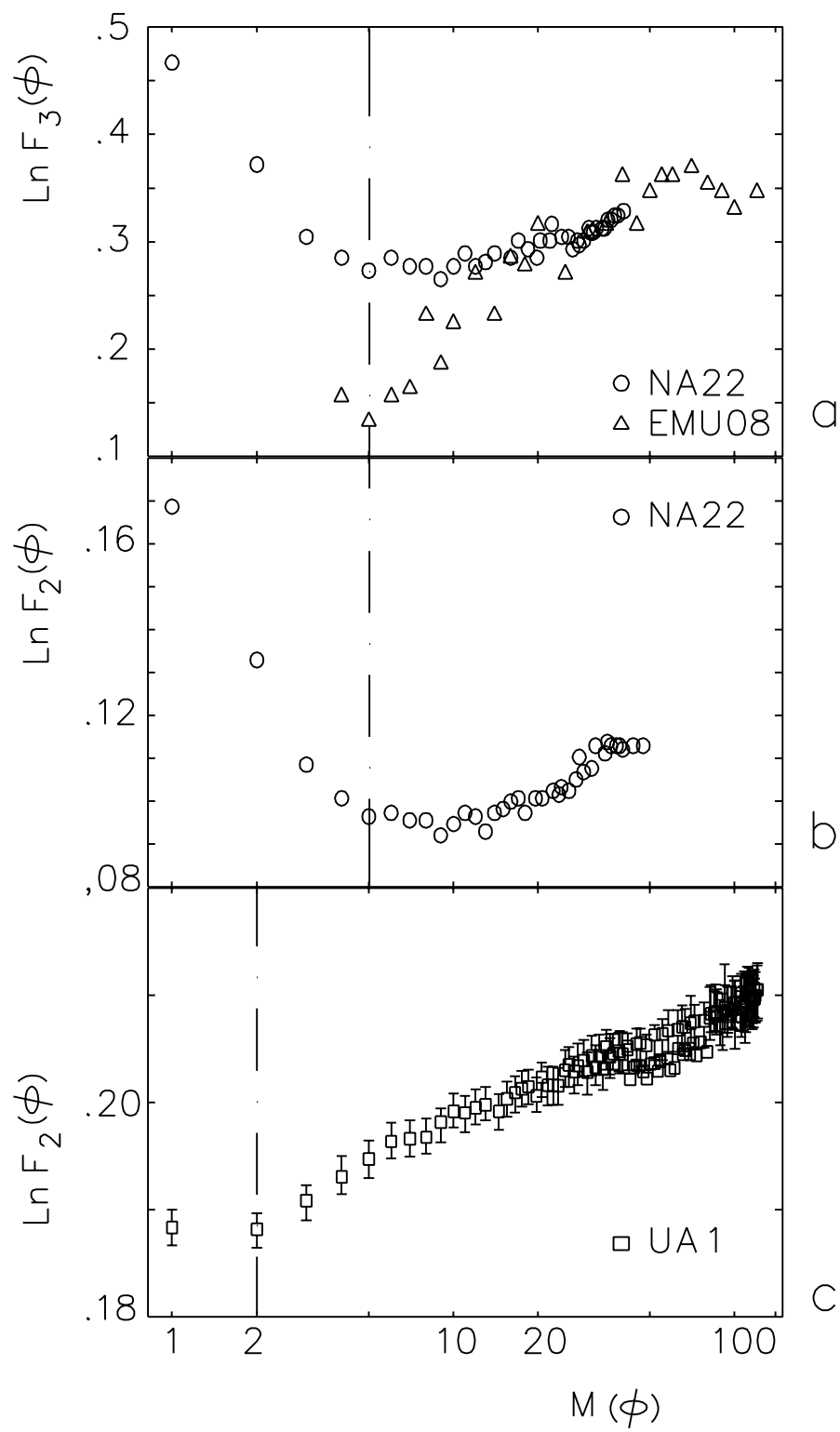


Fig. 1

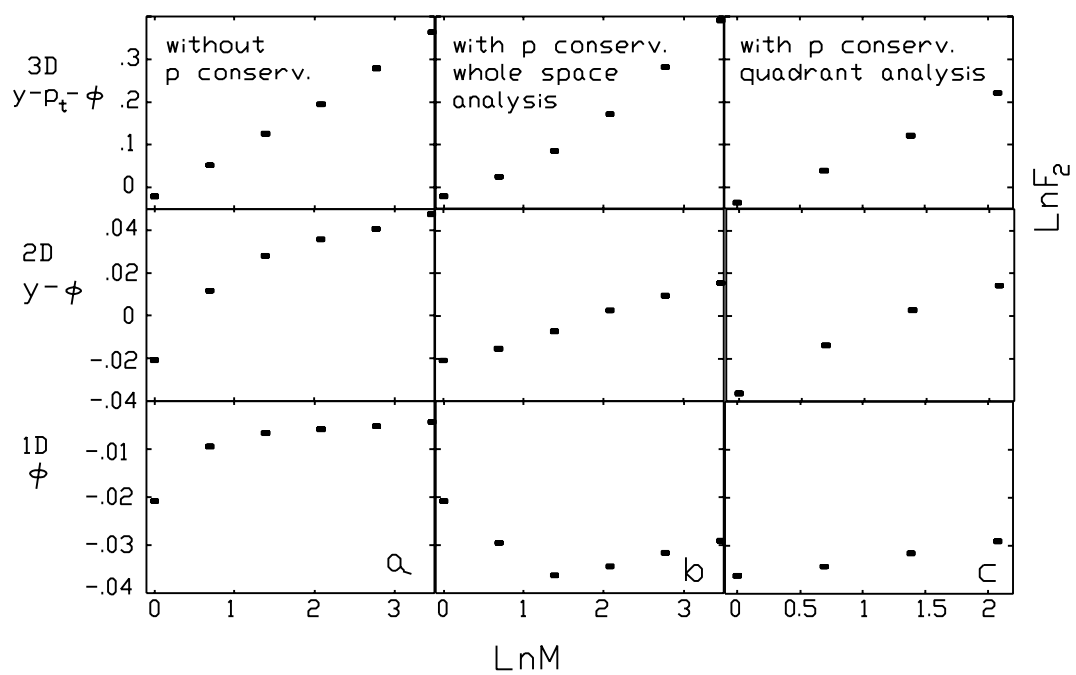


Fig. 5

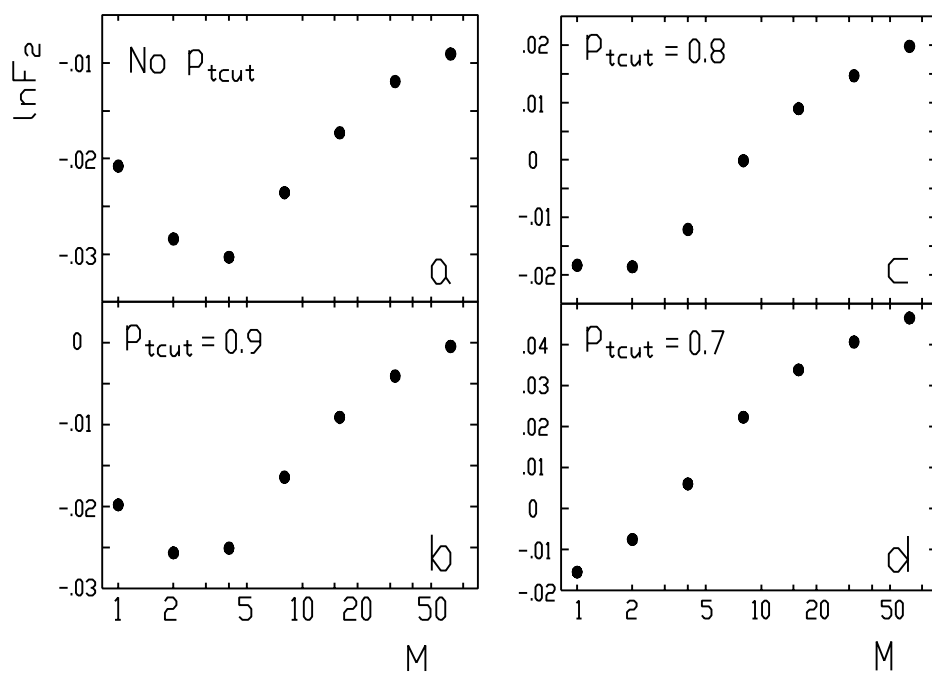


Fig. 6

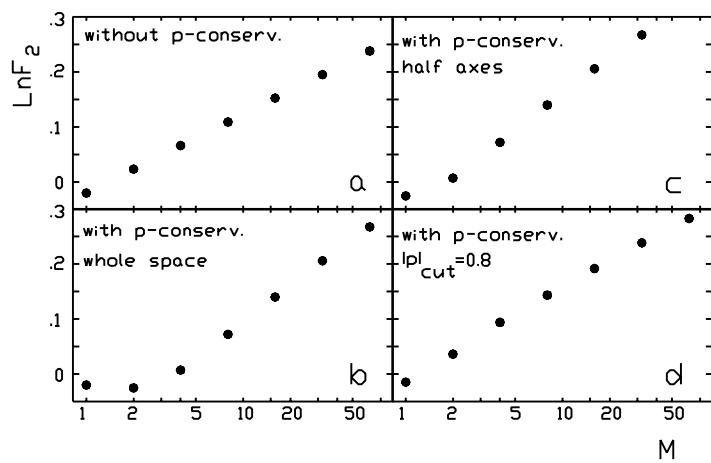


Fig. 2

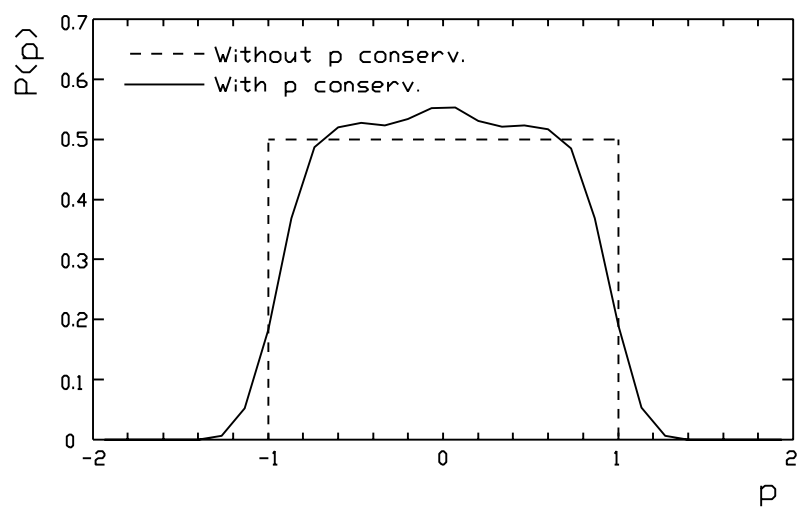


Fig. 3

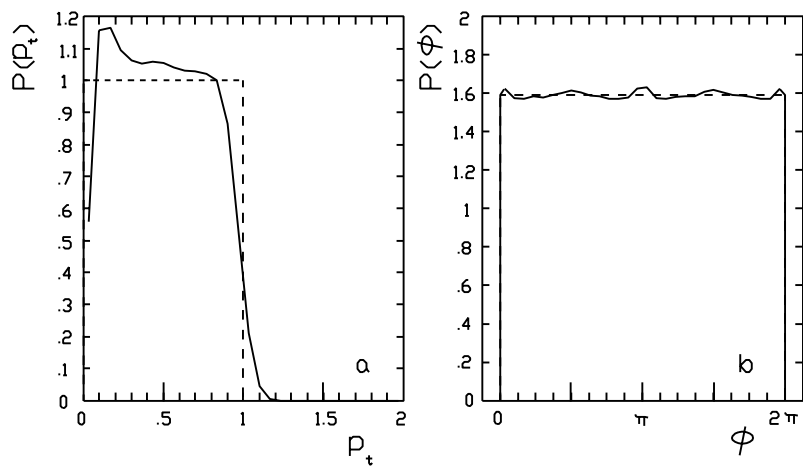


Fig. 4